# Enhancement of SMIB Performance Using Robust Controller

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**Abstract**— This paper represents the use of Robust stabilisation technique to enhance the performance of Single Machine connected to Infinite bus.  $H^{\infty}$  loop shaping Technique is used for the analysis and the controller is developed for the same. The order of the controller obtained is high and thus, model order reduction technique is used to reduce the computational effort. The results are compared with the conventional Power system stabilizer.

Keywords— SMIB, H∞ controller, Order Reduction Technique.

### I. INTRODUCTION

The power system is a highly non linear system whose dynamic performance is influenced by a wide array of devices with different response rate and characteristics. The tendency of the system to develop restoring forces equal to or greater than the disturbing forces is known as stability. The stability problem is concerned with the behaviour of synchronous machines after a disturbance. Small disturbance stability is related to the ability of controlling signals such as voltage, rotor angle and speed etc., following small perturbations such as incremental changes in the system load. This form of stability is determined by the characteristics of load, continuous controls and discrete controls at a given instant of time. Static analysis is done to determine the stability margins and to identify the factors influencing factors.

#### SINGLE MACHINE INFINITE BUS SYSTEM

Synchronous generators form the principal source of electrical energy in power systems. Many large loads are driven by synchronous motors. Power system stability deals with keeping of interconnected synchronous machines in synchronism. Therefore, the characteristics and accurate modelling of the system is very important for stability studies. A mathematical model is developed for the review of steady state and transient performance characteristics. Any disturbance, may be small or big leads to oscillation for which rotor angle stability analysis is significant and the behaviour of rotor dynamics is analysed using swing equation.

A...Swing Equation

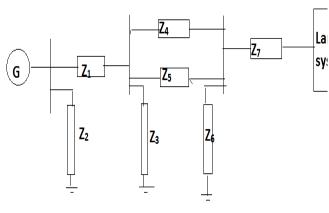
Under normal operating conditions, the relative positions of the rotor axis and the resultant magnetic field axis is fixed. The angle between the two is known as toque or power angle. When a disturbance occurs, the rotor will accelerate or decelerate with respect to rotating air gap MMF, and motion begins. The equation describing this motion is known as swing equation. After the oscillatory period, if the rotors locks back into synchronous speed, the generator will remain in stable state. If the disturbance is due to change in generation, load or any network conditions, the rotor goes to a new operating power angle relative to synchronously revolving field. The swing equation is represented as

$$\frac{H}{180 f_0} \frac{d^2 \delta}{dt^2} = P_m - P_e^{-\dots} (1)$$

Where  $\delta$  is expressed in electrical degrees and H as per unit inertia constant, Pm and P<sub>e</sub> are per unit mechanical and electrical power respectively.

#### **II. SMALL SIGNAL STABILITY OF SMIB**

A general system configuration of SMIB connected to a large system through transmission lines is shown in Fig 1..



 $p\varphi_{fd} = \frac{\omega_{0R_{fd}}}{L_{adu}} E_{fd} - \omega_{0R_{fd}i_{fd}}$ ---(3) Where  $E_{fd}$  is field voltage and  $L_{adu}$  is the mutual flux La linkage.

The state space form including the change in flux is given by  $(\Delta \omega_r)$ 

$$\begin{pmatrix} \Delta \delta \\ \Delta \dot{\varphi}_{fd} \end{pmatrix}^{=} \\ \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & 0 & 0 \\ 0 & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} \Delta \omega_r \\ \Delta \delta \\ \Delta \varphi_{fd} \end{pmatrix} + \begin{pmatrix} b_{11} & 0 \\ 0 & 0 \\ 0 & b_{32} \end{pmatrix} \begin{pmatrix} \Delta T_m \\ \Delta E_{fd} \end{pmatrix} - (4)$$

Where

$$a_{11} = \frac{-K_D}{\frac{2H}{2H}} - (5)$$
$$a_{12} = \frac{-K_1}{\frac{2H}{2H}} - (6)$$

$$a_{13} = \frac{-K_2}{2H} - (7)$$

$$a_{21} = \omega_0 = 2\pi f_0 - (8)$$

$$a_{32} = \frac{-\omega_{0R_{fd}}}{L_{fd}} m_1 L_{ads} - (9)$$

$$a_{33} = \frac{-\omega_0 R_{fd}}{L_{fd}} [1 - \frac{L^1 ads}{L_{fd}} + m_2 L^1 ads] - (10)$$

$$m_1 = \frac{E_{B(X_{Tq} sin\delta_0 - R_T cos\delta_0)}}{D} - (11)$$

$$m_2 = \frac{X_{Tq}}{D} \frac{L_{ads}}{L_{ads} + L_{fd}} - (12)$$

$$b_{11} = \frac{1}{2H} - (13)$$

$$b_{32} = \frac{\omega_0 R_{fd}}{L_{ady}} - (14)$$

#### C.Effects Of Excitation System :

transducer time constant.

The input control signal to the excitation system is normally the generator terminal voltage Er . In the classical generator model, Et is not a state variable. Here, Et is expressed in the terms of other state variables. The state space representation including the excitation system signal perturbations is given by

$$\begin{pmatrix} \Delta \dot{\omega}_{r} \\ \Delta \delta \\ \Delta \phi_{fd}^{\cdot} \\ \Delta \dot{v}_{1} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & 0 \\ a_{21} & 0 & 0 & 0 \\ 0 & a_{32} & a_{33} & a_{34} \\ 0 & a_{42} & a_{43} & a_{44} \end{pmatrix} \begin{pmatrix} \Delta \omega_{r} \\ \Delta \delta \\ \Delta \phi_{fd} \\ \Delta v_{1} \end{pmatrix} + \begin{pmatrix} b_{1} \\ 0 \\ 0 \\ 0 \end{pmatrix} \Delta T_{m} - (15)$$
Where,  

$$a_{34} = \frac{-\omega_{0R_{fd}}}{L_{adu}} K_{A} - (16)$$

$$a_{42} = \frac{K_{5}}{T_{R}} - (17)$$

$$a_{43} = \frac{K_{6}}{T_{R}} - (18)$$

$$a_{44} = \frac{-1}{T_{R}} - (19)$$
Where K<sub>A</sub> is the exciter gain and T<sub>R</sub> is the terminal voltage

#### Fig 1: General Configuration

Analysis of systems having such simple configurations is useful to understand basic effects and concepts. In order to deal with large complex systems an equivalent transmission network is used which is known as single line representation. Developing the expressions for the elements of state matrix as

explicit functions of system parameters is useful in developing the mathematical model of the system under small signal disturbances.

# A. Classical Generator Model:

The generator represented by the classical model is shown in Fig 2. Here  $E^{\setminus l}$  is the voltage behind  $X_d^{-1}$ . Its magnitude is assumed to remain constant at pre-disturbance value .Let  $\delta$  be the angle by which  $E^1$  leads the infinite bus voltage  $E_B$ . As the rotor oscillates during a disturbance,  $\delta$  changes.

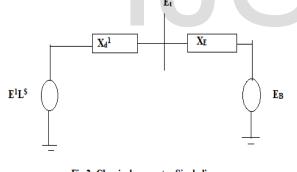


Fig 2 :Classical generator Single line Representation

When the speed deviation  $\Delta \omega_r$  and rotor angle deviation  $\Delta \delta$ are considered the state space representation is

$$\frac{d}{dt} \begin{pmatrix} \Delta \dot{\omega}_r \\ \Delta \dot{\delta} \end{pmatrix} = \begin{pmatrix} -K_D & -K_S \\ 2H & 2H \\ \omega_0 & 0 \end{pmatrix} \begin{pmatrix} \Delta \omega_r \\ \Delta \delta \end{pmatrix} + \begin{pmatrix} \frac{1}{2H} \\ 0 \end{pmatrix} \Delta T_m - (2)$$

Where K<sub>D</sub> is damping torque coefficient, K<sub>S</sub> is synchronizing torque coefficient and H is the inertia constant.

# **B.** Effect Of Field Circuit Dynamics

The field circuit dynamic equation is given by

IJSER © 2015 http://www.ijser.org For the equations shown above , it can be induced that , with positive  $K_5$  the effect of excitation will introduce a negative synchronising torque and positive damping torque component. With negative  $K_5$  positive synchronising and negative damping torque component is introduced for which higher response exciter is beneficial in increasing synchronising torque. But this results in negative damping. An effective way to meet the required exciter performance with regard to system stability is by introducing power system stabilizer.

# **IV.POWER SYSTEM STABILIZER**

The basic function of a power system stabilizer is to add damping to the generator rotor oscillations by controlling its excitation using auxiliary stabilizing signal. To provide damping, the stabilizer must produce a component of electrical torque in phase with rotor speed deviations.

The PSS consists of 3 blocks i) Phase compensation 2)a signal washout signal 3)a gain block.

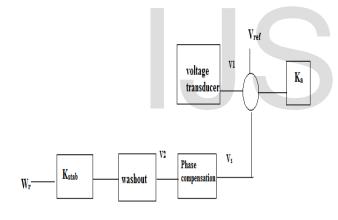


Fig 3: Power system stabilizer

The phase compensation block provides the appropriate phase-lead characteristic to compensate for the phase lag between the exciter input and the generator electrical torque. The signal washout block serves as a high pass filter, with the time constant  $T_W$  high enough to allow signals associated with oscillations in  $w_r$  to pass unchanged. The stabilizer gain  $K_{stab}$  determines the amount of damping introduced by the stabilizer. The state space form including power system stabilizer is given by

$$\begin{pmatrix} \Delta \dot{\omega}_{r} \\ \Delta \dot{\delta} \\ \Delta \dot{\phi}_{fd} \\ \Delta \dot{\nu}_{1} \\ \Delta \dot{\nu}_{2} \\ \Delta \dot{\nu}_{s} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 & 0 \\ a_{21} & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{32} & a_{33} & a_{34} & 0 & a_{36} \\ 0 & a_{42} & a_{43} & a_{44} & 0 & 0 \\ a_{51} & a_{52} & a_{53} & 0 & a_{55} & 0 \\ a_{61} & a_{62} & a_{63} & 0 & a_{65} & a_{66} \end{pmatrix} \begin{pmatrix} \Delta \omega_{r} \\ \Delta \delta \\ \Delta \phi_{fd} \\ \Delta \nu_{1} \\ \Delta \nu_{2} \\ \Delta \nu_{s} \end{pmatrix} - (20)$$

where,  

$$a_{51} = K_{stab}a_{11}$$
--(21)  
 $a_{52} = K_{stab}a_{12}$ --(22)  
 $a_{53} = K_{stab}a_{13}$ --(23)  
 $a_{55} = \frac{-1}{T_W}$   
 $a_{61} = \frac{T_1}{T_2}a_{51}$ ----(25)  
 $a_{62} = \frac{T_1}{T_2}a_{52}$ ----(26)

$$a_{63} = \frac{T_1}{T_2} a_{53} - (27)$$
  

$$a_{65} = \frac{T_1}{T_2} a_{55} + \frac{1}{T_2} - (28)$$
  

$$a_{66} = \frac{-1}{T_2} - \dots - (29)$$

# V.ROBUST CONTROLLER

Power system stabilizer is pertained to certain network configurations. If the configurations change, the system no longer operates in the stability range. In order to overcome this problem, advanced controlling techniques have been developed among which robust stabilisation is one of the modern control techniques.

Robust stability is the minimum requirement of any practical control system. Robust control theory was first developed by Zames and it addresses both the performance and the stability criterion of a control system.

H infinity controlling technique is used for robust stability of the system. Robust  $H_{\infty}$  controllers are developed to provide high robust control environment to linear systems.

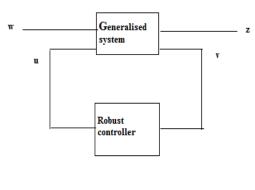


Fig 4:Robust control problem

Considerable advances have been made in the  $H_{\infty}$  control synthesis since it's inception. One can find number of theoretical advantages of this method such as high disturbance rejection, high stability and many more. Mixed weight  $H_{\infty}$ controllers provide a closed loop response of the system according to the design specifications such as model uncertainty, disturbance attenuation at higher frequencies, required bandwidth of the closed loop plant etc. Practically,  $H_{\infty}$  controllers are of high order which, may lead to large control effort requirement. Moreover, the design may also depend on specific system and can require its specific analysis. When  $H\infty$ -optimal control approach is applied to a plant, additional frequency dependent weights are incorporated in the plant and are selected to show particular stability and performance specifications relevant to the design objective as defined.

Various techniques are available in literature for the design of  $H_{\infty}$  controller and  $H_{\infty}$  loop shaping is one of the widely accepted among them as the performance requirements can be embedded in the design stage as performance weights.  $H_{\infty}$  based robust control deals with the characteristics such as amplifiers delay or sensors offset .Considering *G*(*s*) and *K*(*s*) as the open loop transfer function of the plant and controller transfer function respectively, robustness and good performance of closed loop system. Controller *K*(*s*) can be derived, provided it follows three criterions, which are:

1) Stability criterion: If the roots of characteristic equation 1+G(s)K(s)=0 are in left half side of s plane, then stability is ensured.

2)Performance criterion : It establishes that the sensitivity  $s = \frac{1}{1+G(S)K(s)}$  is small for all frequencies where disturbances

and set point changes are large.

3)Robustness criterion : It states that stability and performance should be maintained not only for the nominal model but also for a set of neighbouring plant models that result from unavoidable presence of modelling errors. Robust controllers are designed to ensure high robustness of linear systems. Generally the  $H_{\infty}$  norm of a transfer function F, is the maximum value over the complete spectrum and is represented as

 $||F(jw)||_{\infty} = \sup_{\sigma} (F(jw))$ -(30). Here  $\sigma$  is the singular value of the transfer function. In  $H_{\infty}$  controller synthesis, two transfer functions are used which split complex control problem into separate sections one dealing with stability and the other with performance. The sensitivity function S, and complementary sensitivity function T are used in controller synthesis and is given by

$$S = \frac{1}{\frac{1+GK}{1+GK}} - (31)$$
$$T = \frac{GK}{\frac{1+GK}{1+GK}} - (32)$$

Where G and K are transfer function and controller transfer functions respectively.

From Fig 4, shown above, w is the vector of disturbance signals ,z is the signal positioning of all errors. v is the vector of measurement variables and u is the vector of all control variables. Here the disturbance signal is considered as  $V_{ref}$ . Controller is designed by minimising the norm

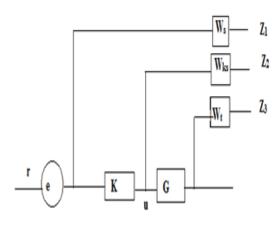
 $min_{k} = ||N(k)||$ ----(33)

$$N = \begin{bmatrix} w_s & S \\ w_T & T \end{bmatrix} - \dots - (34)$$

Where  $W_s$  and  $W_t$  are the weight functions assigned to the problem by the designer. The ultimate objective of the robust control is to minimize the effect of disturbance on output. The sensitivity S and the complementary function T are to be reduced . To achieve this it is enough to minimize the magnitude of |S| and |T| which is done by making  $|S(jw)| < 1/W_s(jw)$  and  $|T(jw)| < 1/W_t(jw)$ .  $W_s$  is the weighting function to limit magnitude of sensitivity function.

 $W_t$  is the robustness weighting function to limit magnitude of complementary sensitivity function .This technique is called as loop shaping technique and is widely used for synthesis of the controller .The shaping objective is to make the output  $y = \Delta \omega$  (Generator frequency variation).

Thus, a stabilising controller k(s) is achieved by minimising cost function y. To obtain the desired frequency response for the plant, loop shaping is employed with the weight functions. There are various methods for loop shaping. The parameters of the weight functions are to be varied so as to get the frequency response of the whole system within desired limits. The block diagram in Fig. 5 describes the mixed Sensitivity problem.



# Fig 5:plant model for $H_{\infty}$ controller

The generalised plant P(s) is given by

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ e \end{pmatrix} = \begin{pmatrix} W_s & -W_s G \\ 0 & W_{ks} \\ 0 & W_t G \\ I & -G \end{pmatrix} \begin{pmatrix} w \\ u \end{pmatrix} --(35)$$

Considering the following state space realizations

$$G^{s} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} - (36)$$

$$W_{s}^{s} = \begin{bmatrix} A_{s} & B_{s} \\ C_{s} & D_{s} \end{bmatrix} - (37)$$

$$W_{ks}^{s} = \begin{bmatrix} A_{ks} & B_{ks} \\ C_{ks} & D_{ks} \end{bmatrix} - (38)$$

$$W_{t}^{s} = \begin{bmatrix} A_{t} & B_{t} \\ C_{t} & D_{t} \end{bmatrix} - (39)$$

$$min \|P\| = min \begin{pmatrix} W_s S \\ W_{ks} KS \\ W_t T \end{pmatrix} = \gamma - (42)$$

Where P is the transfer function w to z. i.e.,  $|T_{zw}|=\gamma$ . Applying minimum gain theorem, we can make the  $H_{\infty}$  norm of  $|T_{zw}|$  less than unity.

$$min \|T_{zw}\| = min \begin{pmatrix} W_s S \\ W_{ks} KS \\ W_t T \end{pmatrix} < 1 - (43)$$

The weights  $W_s$ ,  $W_{ks}$  and  $W_t$  are the tuning parameters and it requires some iterations to obtain weights that yields a good controller . The starting point of weights is given as

$$W_s = \frac{\frac{s}{M} + \omega_0}{s + \omega_0 A} - (44)$$

 $W_{ks} = \text{constant}$  $W_t = \frac{s + \frac{\omega_0}{M}}{As + \omega_0} - (45)$ 

Where A is the allowed steady state offset,  $\omega_0$  is the desired bandwidth and M is the sensitivity peak. Typically, A=0.01 and M=2.

## A. Algorithm to synthesize Robust controller:

1 .Defining the subsystems: In this the open loop transfer function (G) of the system is defined .

2 .Weights are to be selected.

3. Creating the generalised plant.

4. Synthesizing the controller.

5.Reducing the order of controller

6.Obataining the controller transfer function of desired phase margin.

#### VI. RESULTS

For the SMIB system represented in this paper, P=0.9,Q=0.3  $E_t$ =1.0 at 36<sup>0</sup> and  $E_B$  =0.995 Thyristor exciter K<sub>A</sub>=200, T<sub>R</sub>=0.02S PSS added is K<sub>stab</sub>=9.4,T<sub>W</sub>=1.4s,T<sub>1</sub>=0.09s,T<sub>2</sub>=0.3s The constants calculated from the equations are given by

$$K_1=0.7643$$
  $K_2=0.86$   $K_3=0.32$   $K_4=1.41$   $K_5=-0.14$   $K_6=0.416$ 

 $T_3=2.365$   $T_R=0.02$   $K_A=200$ . The transfer function of the lower order robust controller obtained is given by

$$K = \frac{s^8 + 1884s^7 + 8.9e05s^6 + 2.578e06s^5 + 2.494e06s^4 + 1.064e06s^3}{+ 2.05e05s^2 + 1.522e04s + 132.7}$$
  
$$K = \frac{s^9 + 1884s^8 + 8.9e05s^7 + 2.554e06s^6 + 2.503e06s^5 + 1.069e06s^4}{+ 2.054e05s^3 + 1.457e04s^2 - 2.666s - 1.2}$$

$$P = \begin{pmatrix} W_s & -W_s G \\ 0 & W_{ks} \\ 0 & W_t G \\ I & -G \end{pmatrix} = \begin{pmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{pmatrix} -(40)$$

From the equations 35 &40, a mixed sensitivity problem is written as

$$P = \begin{pmatrix} W_s S \\ W_{ks} KS \\ W_t T \end{pmatrix} --(41)$$

In case of mixed sensitivity problem our objective is to find a rational function controller K(s) and to make the closed loop system stable satisfying the following expression

And the reduced form of controller by increasing the desired phase margin is given by

$$K = \frac{66.3S + 148}{S + 148} - (46)$$

The output characteristic using power system stabilizer has settling time of 5.5sec and peak overshoot of 1.5. For the output characteristic using  $H_{\infty}$  controller settling time is 4sec and the peak overshoot is 1.2.

From the results, it is observed that both the settling time and the peak overshoot has reduced for the controller designed using  $H_{\infty}$  loop shaping technique compared with conventional power system stabilizer characteristics.

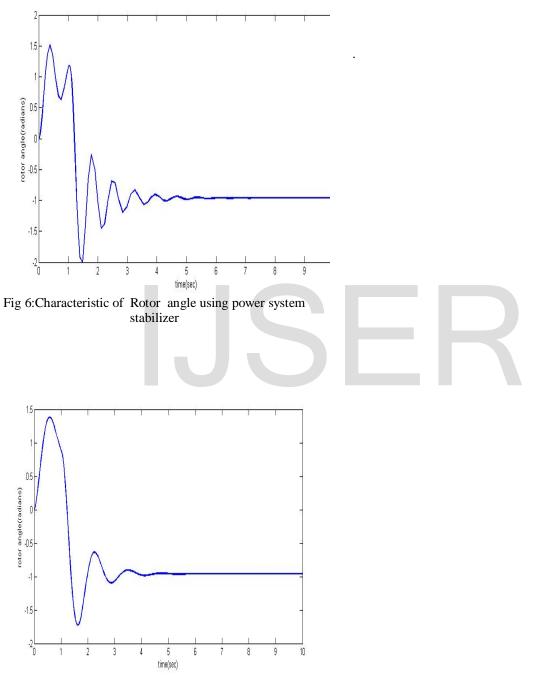


Fig 7: characteristic of Rotor angle using  $H_{\infty}$  controller

#### **VII CONCLUSION**

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